Short Papers

The Capacitances and Surface Charge Distributions of a Shielded Unbalanced Pair

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Abstract—**The capacitance matrix of an unbalanced shielded pair cable is determined theoretically. The wires of the cable are asymmetrically located about the axis of the shield and have different radii; however, the axes of the wires are restricted to lie on a line passing through the axis of the shield.**

The elements of the capacitance matrix are determined as particular elements of the inverse of a truncated infinite matrix, which relates the Fourier coefficients of the surface charge densities on the inner conductors and the shield to the applied voltage excitations on the cable conductors.

The capacitances and surface charge distributions are evaluated numerically for a shielded pair cable, which, due to inaccuracies in the cable manufacturing processes, has one wire with a smaller or larger radius than the other wire of the pair and/or has one wire closer to or farther from the axis of the shield than the other wire of the pair.

I. Introduction

In the earlier papers on this subject [1]–[3], only the various capacitances associated with the balanced cable structure, e.g., the direct, ground, and mutual capacitances, were determined. These capacitances were determined indirectly from a consideration of only the case of "balanced" or "longitudinal" excitation, without directly calculating the Fourier coefficients of the various surface charge densities involved. In a later paper [4], all of the Fourier coefficients of the various surface charge densities were also calculated.

In this short paper, the previous methods for determining the elements of the capacitance matrix are extended to an unbalanced shielded pair cable, i.e., to a cable in which the wires are asymmetrically located about the axis of the shield and have different radii. In the process, the Fourier coefficients of the various surface charge densities and the various capacitances associated with the unbalanced cable structure are determined. The voltage excitations on the inner conductors and the shield are assumed to be completely arbitrary, but the axes of the wires are restricted to lie on a line passing through the axis of the shield.

II. Cable Geometry

The geometry of the cable is shown in Fig. 1. The cable consists of two straight spatially separated cylindrical inner conductors embedded in a simple insulator (i.e., a linear, homogeneous, isotropic, and time invariant medium), and is enclosed by a conducting annular shield. It is assumed that the centers of the wires are spaced unequally at distances s' and s'' on the same line from the center of the shield. It is assumed that the wires are of different circular cross sections with radii δ' and δ'', and are composed of the same conducting materials. The dielectric in which the wires are embedded completely surrounds each wire and extends uniformly out to the inner radius Δ of the annular shield. The dielectric is determined by its constitutive parameters ε and μ. It is assumed that the conductivities of each wire and the shield are infinite and that conductivity of the dielectric is zero.

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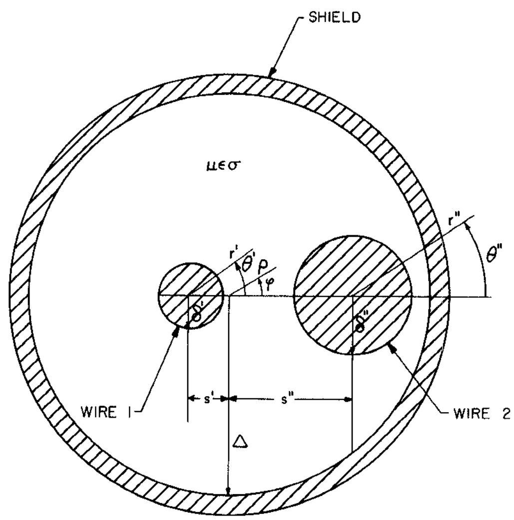

Fig. 1. Cross section of an unbalanced shielded pair cable.

Dimensional restrictions are imposed on the parameters of the cable to keep the three conductors of the structure from touching, i.e., let

$$s > \delta$$

$$\Delta > \max(s' + \delta', s'' + \delta'')$$

where

$$s \equiv s' + s''$$

$$\delta \equiv \delta' + \delta''.$$

III. Theory

With the aforementioned assumptions, the surface charge densities on each wire and the shield are functions of only the azimuthal angles describing the circumferences of these conductors and are expanded in Fourier series in these azimuthal angles, with the various radii of the wires and the shield as parameters. The resulting electrostatic potential distributions due to these surface charge distributions are also represented in Fourier series in these azimuthal angles. The total electrostatic potential distribution within the cable structure is obtained by superposition, after several coordinate transformations are made to force all of the Fourier series into a common representation.

The Fourier coefficients of the electrostatic potential distribution are related by Laplace's equation to the Fourier coefficients of the surface charge densities on the inner conductors and the shield. The solution of Laplace's equation, subject to the boundary conditions impressed by the applied voltage excitations on the cable conductors, gives rise to an infinite matrix which relates the Fourier coefficients of the surface charge densities to the applied voltage excitations on the various conductors.

Exact expressions for the elements of the capacitance matrix are then determined as particular elements of the inverse of the infinite matrix. If the wire radii are small relative to the wire spacings and if the wire spacings are small relative to the shield

TABLE I
CAPACITANCE MATRIX (farads per meter)

	(nominal) 0%	Case 1 (changes in δ)		Case 2 (changes in s)		Case 3 (changes in $\binom{\delta}{s}$)			
		−10%	+10%	−10%	+10%	=10%	∓10%	±10%	∓10%
C_{11}	0.536(−10)	0.527(−10)	0.547(−10)	0.552(−10)	0.524(−10)	0.540(−10)	0.517(−10)	0.566(−10)	0.533(−10)
C_{12}	−0.154(−10)	−0.140(−10)	−0.170(−10)	−0.174(−10)	−0.138(−10)	−0.157(−10)	−0.126(−10)	−0.194(−10)	−0.151(−10)
C_{21}	−0.154(−10)	−0.140(−10)	−0.170(−10)	−0.174(−10)	−0.138(−10)	−0.157(−10)	−0.126(−10)	−0.194(−10)	−0.151(−10)
C_{22}	0.536(−10)	0.487(−10)	0.591(−10)	0.536(−10)	0.544(−10)	0.487(−10)	0.492(−10)	0.591(−10)	0.601(−10)
C_d	0.154(−10)	0.140(−10)	0.170(−10)	0.174(−10)	0.138(−10)	0.157(−10)	0.125(−10)	0.193(−10)	0.151(−10)
C_g 1	0.382(−10)	0.387(−10)	0.378(−10)	0.378(−10)	0.387(−10)	0.383(−10)	0.391(−10)	0.374(−10)	0.382(−10)
C_g 2		0.347(−10)	0.421(−10)	0.362(−10)	0.406(−10)	0.330(−10)	0.367(−10)	0.397(−10)	0.449(−10)
C_m	0.345(−10)	0.323(−10)	0.369(−10)	0.359(−10)	0.336(−10)	0.335(−10)	0.314(−10)	0.385(−10)	0.358(−10)

radius, then accurate numerical approximations for the elements of the capacitance matrix are obtained to any degree of accuracy by suitably truncating the infinite matrix.

The derivation of the unbalanced pair closely parallels the derivation of the balanced pair [4], and, therefore, the details of the derivation are not given here. However, a complete derivation is available in the References [5].

IV. RESULTS

The preceding theoretical results are now applied to determine the Fourier coefficients of the surface charge densities on the inner conductors and the shield and the various capacitances associated with the cable structure for a realistic cable geometry with various impressed voltage excitations. In particular, both "balanced" and "longitudinal" voltage excitations on a typical standard production cable are considered. The nominal geometrical parameters for this cable are

$$s' = s'' = 25.149 \text{ mils}$$

$$\delta' = \delta'' = 12.587 \text{ mils}$$

$$\Delta = 73.175 \text{ mils.}$$

This cable is an equivalent shielded pair model for one pair of a 50 pair PIC cable manufactured by Western Electric.

The effective relative permittivity of an equivalent uniform dielectric surrounding the wires, as determined indirectly from a method based on previous capacitance measurements, is

$$\varepsilon_r = 2.026.$$

In order to determine the separate effects on the elements of the capacitance matrix of a change in the wire size alone (case 1) or a change in the wire separation alone (case 2), in addition to the combined effects of a simultaneous change in the wire size and the wire separation (case 3), three cable geometries are considered.

Case 1: One of the wires of the pair either has a 10 percent smaller or larger radius than the other wire of the pair.

Case 2: One of the wires of the pair is either 10 percent closer to or farther from the axis of the shield than the other wire of the pair.

Case 3: One of the wires of the pair has either a 10 percent smaller or larger radius and is either 10 percent closer to or farther from the axis of the shield than the other wire of the pair.

In all of these cases, the size of wire 1 and the spacing of wire 1 from the axis of the shield are held fixed, and only the size of wire 2 and the spacing of wire 2 from the axis of the shield are varied.

Table I contains the values of the capacitances to ground C_{g1} and C_{g2}, the direct capacitance C_d, and the mutual capacitance C_m, in addition to the four elements of the capacitance matrix C_{ij} $(i,j = 1,2)$, for the modified cable geometries described previously. These capacitances are related by

$$C_m = C_d + \frac{C_{g1}C_{g2}}{C_{g1} + C_{g2}}$$

where

$$C_d = -C_{12} = -C_{21}$$

and

$$C_{g1} = C_{11} + C_{12}$$

$$C_{g2} = C_{22} + C_{21}.$$

Figs. 2–7 contain plots of the surface charge densities on the inner conductors (labeled wires 1 and 2) and the shield, as constructed from their Fourier coefficients, for the modified cable geometries and the various voltage excitations described previously.

V. DISCUSSION

The capacitance differences shown in Table I are to be expected due to the proximity effects of changing the wire spacings and wire radii. Obviously, the direct capacitance of the wire with the plus (minus) 10-percent deviation in its radii and the minus (plus) 10-percent deviation in its spacing should be larger (smaller) than the direct capacitance of the wire with the other combinations of the wire parameters, as shown in Table I. Also, the effects on the direct capacitance of increasing (decreasing) the wire radius and simultaneously increasing (decreasing) the wire spacing tend to cancel, as shown in Table I. Similarly, the capacitance to ground of the wire with the plus (minus) 10-percent deviation in its diameter and the plus (minus) 10-percent deviation in its spacing should be larger (smaller) than the capacitance to ground of the wire with the other combinations of the wire parameters, as shown in Table I. Also, the effects on the capacitance to ground of increasing the wire radius and simultaneously decreasing the wire spacing, or vice versa, tend to cancel, as shown in Table I.

The capacitance values presented are useful in determining the maximum capacitive unbalance (per unit length) that can be

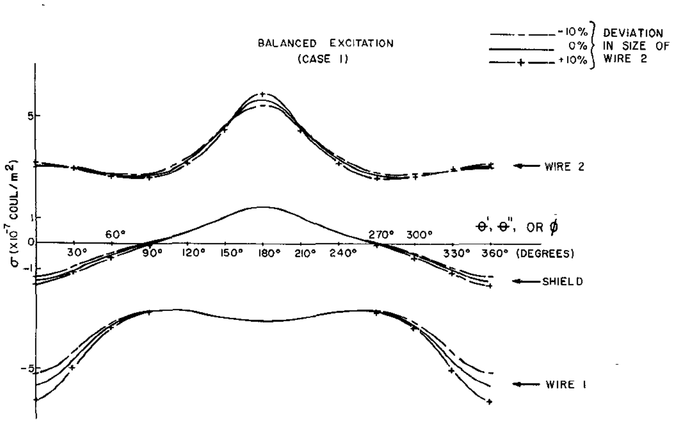

Fig. 2. Surface charge density versus azimuthal angle/balanced excitation (case 1: changes in radius).

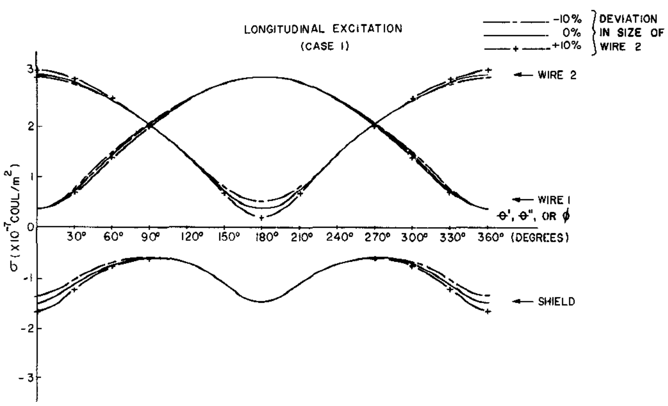

Fig. 3. Surface charge density versus azimuthal angle/longitudinal excitation (case 1: changes in radius).

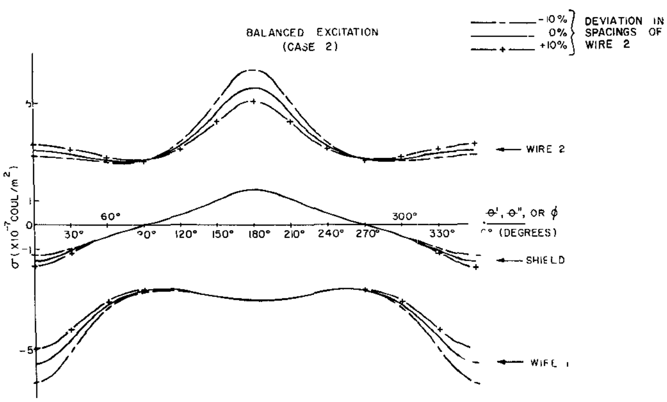

Fig. 4. Surface charge density versus azimuthal angle/balanced excitation (case 2: changes in spacing).

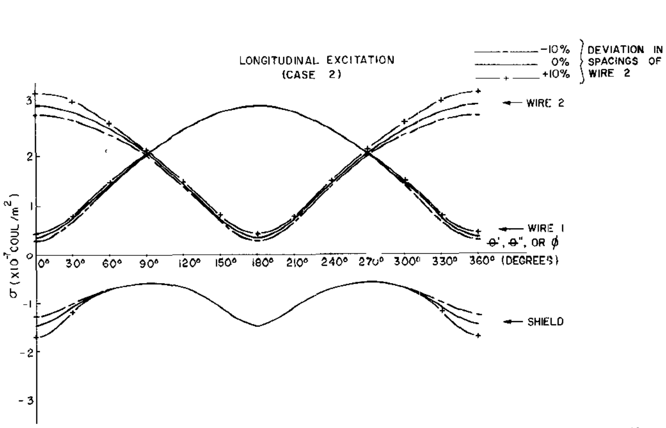

Fig. 5. Surface charge density versus azimuthal angle/longitudinal excitation (case 2: changes in spacing).

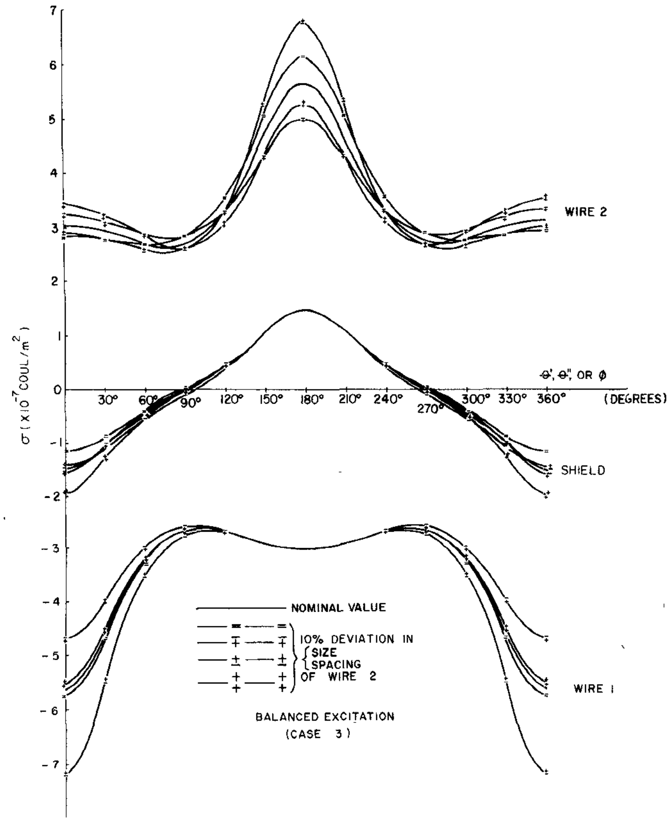

Fig. 6. Surface charge density versus azimuthal angle/balanced excitation (case 3: changes in radius and spacing).

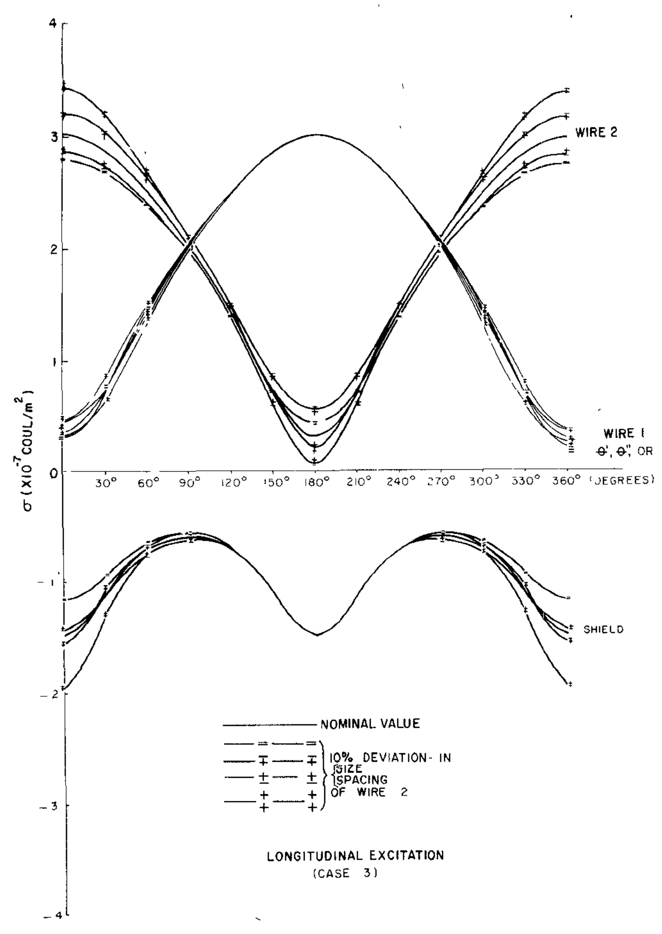

Fig. 7. Surface charge density versus azimuthal angle/longitudinal excitation (case 3: changes in radius and spacing).

expected from a production cable. For the aforementioned cases, the worst unbalance observed was 0.95 pF/m when one diameter was increased by 10 percent over its nominal size and moved 10 percent farther away from the axis of the shield than its nominal position. The Fourier coefficients of the surface charge densities (not presented, cf. [5]) are required in determining the propagation constants and associated propagation modes that can be expected on a production cable.

VI. Conclusion

In this short paper the capacitance matrix of a straight pair of wires in a shield was determined theoretically. The Fourier coefficients of the surface change densities on the inner conductors and the shield and the various capacitances associated with the cable structure were then determined. The voltage excitations were completely arbitrary and the cable structure was unbalanced in the sense that the wires can be asymmetrically located about the axis of the shield and can have different radii. The theoretical results were evaluated numerically for the case of a shielded pair cable modified to reflect possible inaccuracies in the nominal size and spacing of the wires due to inaccuracies in the manufacturing process.

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Equivalent Circuits and Characteristics of Inhomogeneous Nonsymmetrical Coupled-Line Two-Port Circuits

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***Abstract*—The equivalent circuits and the properties of nonsymmetrical coupled-line two-port prototype circuits in an inhomogeneous medium are presented. Potential applications include both symmetrical and nonsymmetrical circuits for narrow- and wide-band applications as filters and impedance transformers.**

Introduction

Symmetrical and nonsymmetrical two-port circuits consisting of nonsymmetrical uniformly coupled lines in an inhomogeneous medium [1] may be designed for various applications as filters and impedance matching networks by utilizing the coupled-line four-port parameters and resulting equivalent circuits for various structures. Equivalent circuits and some characteristics of identical coupled lines in an inhomogeneous medium have been obtained by Zysman and Johnson [2]. Allen [3] has utilized these to formulate the design procedures for various two-port circuits for application as filters for the case of large mode velocity ratios. For the general case of lossless quasi-TEM coupled lines in an inhomogeneous medium, the four-port equivalent circuit in a graph equivalent form [4] has been obtained by Costamagna and Maltese [5], and the equivalent circuit consisting of uncoupled normal mode lines with mode decoupling and coupling networks consisting of ideal transformer banks has been given by Chang [6]. In addition, the equivalent circuits for various prototypes for the special case of congruent nonsymmetrical coupled lines, where the line parameters are such that the normal modes degenerate into an even-voltage and an odd-current mode [1], have been derived by Allen [7]. In this short paper, the properties of nonsymmetrical coupled lines in an inhomogeneous medium and their four-port parameters as obtained by Tripathi [1] are used to obtain the equivalent circuits and characteristics of various prototypes for the case of quasi-TEM lossless lines. Such structures offer inherent impedance transforming capability and added flexibility in design through an additional variable as compared to identical coupled lines in an inhomogeneous medium.

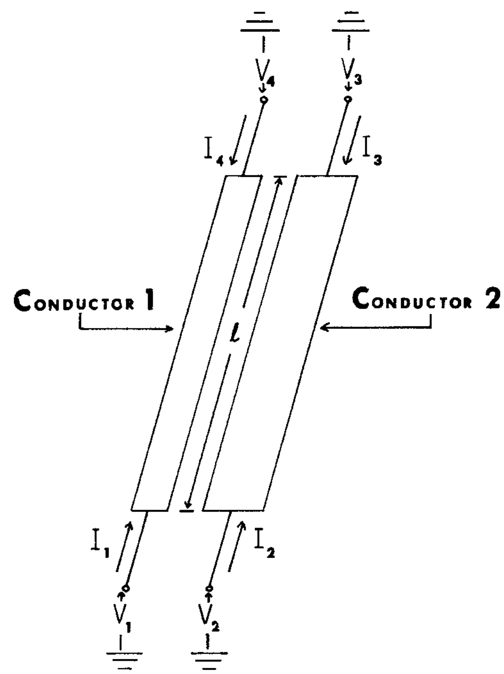

Fig. 1. Schematic of a nonsymmetrical coupled-line four-port in an inhomogeneous medium.

The immittance matrix elements for the coupled-line four-port (Fig. 1) have been derived in terms of the normal mode parameters of the coupled system [1]. These parameters are the propagation constants, the mode voltage ratios on the two lines, and the partial mode impedances and admittances of the two lines for the normal modes of the coupled system. For the case of lossless coupled lines, characterized by their self- and mutual inductances per unit length and capacitances per unit length as given by L_1,L_2,L_m and C_1,C_2,C_m where L_j and C_j ($j = 1,2$) are self-inductance and capacitance per unit length of line j in presence of line k ($k = 1,2$: $k \neq j$), and L_m and C_m are mutual

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